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ABSTRACT

Abstracts of 12 mathematics education research reports and critical comments (by the abstractors) about the reports are provided in this issue of Investigations in Mathematics Education. The reports are: "More Precisely Defining and Measuring the Order-Irrelevance Principle" (Arthur Baroody); "Children's Relative Number Judgments: One-to-One Correspondence, Recognition of Non-Correspondence, and the Influence of Cue Conflict" (Richard Cowan); "Intuitions on Functions" (Tommy Dreyfus and Theolore Eisenberg); "The Effects of Time and Data Permutation on the Solution Strategies of First Grade Children" (Alan Dudley); "Factors Affecting Achievement in the First Course in Calculus" (Orlyn Edge and Stephen Friedberg); "Does the Teaching of Probability Improve Probabilistic Intuitions?" (E. Fischbein and A. Gazit); "Monitoring the Mathematics Achievement of Black Students" (Lyle Jones, Nancy Burton, and Ernest Davenport); "Absence of a Sex Difference in Algorithms for Spatial Problem Solving" (Robert Kail, Michael Stevenson, and Katherine Black); "Sex Differences in Children's Mathematics Achievement: Solving Computations and Story Problems" (Sandra Marshall); "Computational Estimation and Related Mathematical Skills" (Rheta Rubenstein); "Computational Estimation Procedures of School Children" (Judith Threadgill-Sowder); and "Numbers in Contextual Frameworks" (F. van den Brink). References to mathematics education research in CIJE (by EJ number) and RIE (by ED number) from January through March 1985 are also included. (JN)



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Volume 18, Number 3 - Summer 1985

THE ERIC SCIENCE, MATHEMATICS AND **ENVIRONMENTAL EDUCATION CLEARINGHOUSE** in cooperation with Center for Science and Mathematics Education

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INVESTIGATIONS IN MATHEMATICS EDUCATION

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Baroody, Arthur J. MORE PRECISELY DEFINING AND MEASURING THE ORDER-IRRELEVANCE PRINCIPLE. <u>Journal of Experimental Child Psychology</u> 38: 33-41; August 1984.

Abstract and comments prepared for I.M.E. by JAMES HIEBERT, University of Delaware.

1. Purpose

The study is designed to demonstrate that there are two distinct components of Gelman's and Gallistel's (1978) order-irrelevance principle in young children's counting: (1) objects can be counted in any order, and (2) the order in which the objects are counted does not affect the total.

2. Rationale

Adult logic suggests that the two components of counting objects in different orders and knowing that the counts will yield the same answer are interchangeable; understanding one of the components implies understanding the other. Gelman and Gallistel (1978) used tasks that measure the first component and inferred an understanding of both from success on the first. Based on related previous data, the author of this study hypothesizes that the two are distinguishable and that the first component, termed order-indifference, is a developmentally less advanced notion than the second, order-irrelevance. Presumably, young children are willing to count a set of objects in different orders even though they do not realize that they will get the same total.

3. Research Design and Procedures

The subjects were 45 kindergarten and 56 first-grade children in two rural community schools. The children were interviewed individually. Each child was presented with a linear array of eight objects and asked a series of four questions: (a) "How many are there?"; (b) "Could you



make this number 'one' (interviewer pointing to the last item counted by the child) and count the other way?"; (c) "We got N {the number given by the child in (a)} counting this way—what do you think we would get counting this way?" The interviewer's question was accompanied by pointing gestures indicating the reverse direction from that used by the child, and the array was covered to prevent further counting; (d) the child was then asked to recount the array in the new direction.

Question (a) was used to screen subjects from the study. Questions (b) and (d) were adopted from Gelman and Gallistel (1978) and were used to assess order-indifference. Subjects were scored as successful on (b) if they indicated verbally or nonverbally that the end item could be labeled "one". Subjects were scored as successful on (d) if they counted in the direction opposite to their first count and gave an answer of 8±1. Subjects were given credit for order-indifference, for being willing to disregard order in their counting, if they succeeded on both (b) and (d).

Ouestion (c) was used to assess children's understanding of the order-irrelevance principle. Subjects were scored as successful if they gave the same number as they had given to question (a).

4. Findings

All subjects counted correctly (+1) in response to question (a) so none were excluded from the study. All subjects were successful on question (b) and all but one kindergarten subject were successful on question (d). Thus, 100% of the first graders and 98% of the kindergarteners were credited with order-indifference.

For question (c), 13 kindergarteners gave an incorrect prediction, eight indicated that they did not know the answer, three indicated that they were unsure and then gave an incorrect prediction, and one



indicated he was unsure and then counted. The totals presented in the table indicate that one additional subject was unsuccessful, but the author does not describe this subject's behavior. Five first graders gave incorrect predictions and three indicated they were unsure and then gave an incorrect prediction. Thus, 45% of the kindergarten subjects and 87% of the first-grade subjects were credited with order-irrelevance, with understanding that the order in which objects are counted does not affect the cardinal number of the set.

5. Interpretations

The author concludes that there is a need to distinguish between order-indifference and order-irrelevance. The results are believed to indicate a developmental trend: children first utilize order-indifference in their counting and later discover the implications of their actions and recognize the order-irrelevance principle.

Abstractor's Comments

The study raises two issues of special interest: how counting skills develop, and how we assess how counting skills develop. Regarding the first, it is clear once again that the development of mathematical behaviors is a complex process. It also is clear, in a rather striking way, that adult logic is of little help in sorting out the complexities. Apparently some young children believe that it is acceptable to count objects in two different orders without knowing that they will get the same answer either way.

Although such behavior might be unexpected, there are some interesting parallels between young children's actions in this study and the behavior of older students who have had years of instruction. It is well known that many students learn to carry out a mathematical procedure without knowing the reason for the procedure or why it works. In some settings students are willing to believe that the same problem



can have different answers since different methods were used to solve it. A plausible explanation for this behavior is that students have memorized symbol manipulation rules without understanding them and thus they are unable to recognize the conceptual similarities between the methods. Like their older counterparts, the children in this study seem to feel that they can get different answers and both will be correct because they have used different "methods." Without being able to trace the problem to instructional inadequacies, it is more difficult to explain this counter-intuitive feature of young children's developing counting skills.

Perhaps the most interesting contrast between the interpretations of this author and those of Gelman and Gallistel concern the developmental order of skill and understanding. Although not stated explicitly, the author implies that order-indifference is a feature of the counting skill, whereas order-irrelevance is a courting principle that represents a fundamental understanding of the counting act. The author's data suggest that the skill is perfected before the understanding is acquired. Gelman and Gallistel claim that the opposite is true. Of course, major issues such as this are not decided on the basis of a single set of results and the interpretation of the crder-indifference and order-irrelevance constructs is not yet settled, but the contrasting viewpoints are interesting nonetheless.

The second major point of interest is what the study says about assessing children's competencies. The author suggests that Gelman and Gallistel infer too much about children's competencies from their tasks, and demonstrates that different tasks generate different performance. The question for the reader is what competencies or incompetencies can be inferred from performance on the author's tasks.

Changes in task context are known to produce substantial changes in performance. Of special interest here is the use of language in asking questions. Is it possible that the children may have interpreted



question (c) ('We got N counting this way--what do you think we would get counting this way?") as a challenge to their initial count or as an invitation to change their answer (and this time get it right)? The way in which questions are phrased does affect young children's responses (Donaldson, 1978). Braine and Rumain (1983) describe this phenomenon by distinguishing between ordinary comprehension, determining the intended meanings in ordinary comprehension, determining the intended meanings in ordinary comprehension, and analytic comprehension, determining the literal meaning -- what the language says rather than what the speaker usually means by it. They point out that many laboratory tasks require analytical comprehension whereas young children are more familiar with and better at ordinary comprehension. These ideas may muddy the interpretation of children's responses rather than clarify them, but they remind us of the caution that must be used when inferring competencies and incompetencies from performance on a small set of tasks.

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Cowan, Richard. CHILDREN'S RELATIVE NUMBER JUDGMENTS: ONE-TO-ONE CORRESPONDENCE, RECOGNITION OF NON-CORRESPONDENCE, AND THE INFLUENCE OF CUE CONFLICT. Journal of Experimental Child Psychology 38: 515-532; December 1984.

Abstract and Comments prepared by J. PAUL MCLAUGHLIN, Purdue University-Calumet, Hammond, Indiana.

1. Purpose

This article reports and discusses the results of four experiments related to relative number judgments made by five-year-old children in England. The experiments were designed to give insight into how, or on what basis, children make decisions about which of two sets has the greater number of objects. What cues do the children follow when asked to tell which set has more objects: "...the question arises of whether it is appropriate for psychologists, as opposed to mathematicians, to consider the processes of one-to-one correspondence involved in counting and relative number judgments to be related" (p. 516).

2. Rationale

There is disagreement over how well children understand one-to-one correspondence. Part of the problem arises from the fact that one-to-one correspondence is used in counting situations as well as in comparison situations. Disagreement also exists regarding the links between the one-to-one correspondence component of counting, relative number judgments, and number conservation.

On what basis does a child make a choice of the column which has more dots? Is the basis the overall density of dots, the length of the column, the presence of "gaps" or blanks in a column, or the results of a one-to-one pairing between the columns? Several references are made to the work of different theorists which give conflicting answers to parts of this question.



In these experiments the researcher was interested in the effect of perceptual cues such as density, column length, "gaps," and the presence of guidelines on a child's relative number judgment. (By guidelines is meant lines on the picture showing a one-to-one pairing of elements in one set with elements in another set.) Which cues have an effect and, if conflicting cues are present, which one predominates? No formal hypotheses are stated, but the rationale for experiments 2, 3, and 4 is given in terms of the results of previous experiments.

3. Research Design and Procedures

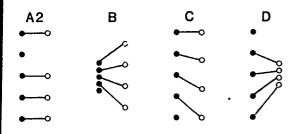
In each of the four experiments, children were asked to make relative number judgments about pairs of sets presented as columns of colored dots. Relative number judgment meant deciding which of two non-equivalent sets has more objects. The subjects were five-year-old children, mean age 5 years 6 months, attending infant school in England. A different group of 18 to 24 children was used for each of the experiments. In most cases the child spoke English as a first language.

The materials consisted of sets of large cards (30 cm x 21 cm) with two columns of dots of uniform size. The two columns differed in color, and for each display type there were two identical cards, except that colors were reversed. Color was found to have no influence on the subjects' responses. Children identified their column choice by color name.

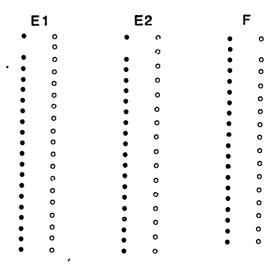
The displays on the cards used in the experiments are shown and identified (see Figure 1).

In experiment 1 "small number" cards with 4 or 5 dots per column and "large number" cards with 19 or 20 dots per column were used. Only large number cards were used in experiments 2, 3, and 4. Display F had only 16 dots, equally spaced, in the less numerous column. All other displays had 19 dots in the less numerous column. The number of dots





Small Numbers With Lines (Not to Scale) (p. 521)



Large Numbers Without Lines (Not to Scale) (p. 526)

Figure 1. Display types.



on the large number cards was judged too many for the subjects to quantify.

In each experiment children were tested individually. A pretest was given to ensure that each subject could distinguish and name the colors used and knew that 3 is more than 2. In scoring, one point was given each time the subject gave the correct response to the question, "Are there more blue dots or more red dots?". Decks were shuffled and presented three or four times. Scores were tabulated by display type.

In experiment 1 the effect of guidelines was studied. Displays A1, B, C, and D, with and without guidelines, and with both large numbers and small numbers, were used. Means and standard deviations for each of the sixteen cells are given and F-values for statistically significant main effects and interactions are included in the report.

In experiment 2 only large number A2 cards without guidelines were used. The mode of presentation of the card varied from showing the entire card to the subject, to showing each column separately, to showing the top and bottom halves separately. A direct one-to-one pairing was precluded in modes 2 and 3. The percent of correct responses for each mode of presentation is given.

In experiment 3 large number display cards A2, E1, E2, and F were used. The question investigated was, "Is the gap-cue consistently followed?" Means, standard deviations, and F-values from one-way analysis of variance are reported.

In experiment 4 displays A2, B, C, and E1 with and without guidelines were used to answer the question, "What happens when there is a cueconflict?" The mean number of correct responses per subject are reported for each of the eight cells. The change in scores is compared for the different display types using a t-test.



4. Findings and Interpretations

From experiment 1, when making comparisons involving small numbers of dots in columns of equal length, the scores were high (75% to 100%) with or without guidelines. For small number displays with columns of unequal length, scores increased from 30% without guidelines to 70% with guidelines. Children apparently can quantify and compare "how many" for small numbers of objects; however, if columns are not of the same length, then the longer column is more often chosen when guidelines are not present.

For large numbers of dots children c .not determine how many are in a column. Hence, they base their relative number responses on other cues. The presence of guidelines on displays A2 and D had little effect on the already high (90% +) scores without guidelines. Scores on display B remained low--2% without guidelines, 25% with guidelines. Experiment 4 also explored the use of guidelines on displays A2, B, and C.

Experiment 2 investigated whether the gap was the cue on display A2. Correct responses ranged from 86% to 94% for all three modes of presentation. Since the one-to-one pairing was precluded in presentation modes 2 and 3, it appears that the gap-cue rather than the one-to-one pairing is operative.

This conjecture gained further support in experiment 3 in which only large number displays with a gap in one of the columns were used. Subjects had average scores of 8.1 and 8.6 out of 12 on the two displays (A2 and F) in which the less numerous column had a gap and the more numerous column did not have a gap. In contrast, the average was 4.3 out of 12 correct on both of the displays (E1 and E2) in which the more numerous column had a gap and the less numerous column did not have a gap. It appears that subjects followed the gap-cue as opposed to the overall (average) density-cue about 65% of the time.



Experiment 4 examines what happened when the guideline-cue conflicted with either the gap-cue or the length-of-column-cue. Three of the display types had been used before in experiment 1 with other children. The results were the same before. On display A2, 80% were correct without guidelines and 90% with guidelines; on display B, 10% correct without guidelines and 20% correct with guidelines; on display C, 40% correct without guidelines and 80% correct with guidelines. On display E1, which had not been used with guidelines, results were 20% correct without guidelines and 65% correct with guidelines.

On display A2 there was no conflict between the gap-cue and the guideline-cue and not much change in scores. On display B there was a conflict between the length-cue and guideline-cue. In this case it appears that the length-of-column-cue was stronger than the guideline-cue. That is, the longer column was chosen as having more dcts regardless of what the guidelines indicated. On display C there was a possible density-cue; however, the guideline-cue predominated. Likewise on display E1 the gap-cue and the guideline-cue conflicted; however, subjects followed the guideline-cue.

Adding guidelines which conflict with a gap-cue gave statistically significant greater improvement in scores when compared to adding guidelines which conflict with a length-of-column-cue. In general, guidelines help in making correct decisions when columns are of equal length. Guidelines are less helpful when columns are of unequal length.

Further research could involve children of other ages. Also, since only one-to-one correspondences were investigated, further research using many-to-one correspondences is another direction for future research.

Abstractor's Comments

My reaction is that this is a worthwhile report in its own right, and provides another entry in the "Needed Research" catalog. While the



age of the children and the content may appeal to a smaller group, the broader purpose of gaining insight into how children develop knowledge should appeal to mathematics educators of all levels. The research is psychological, by a psychologist who sounds this precaution: "A third practice . . . is allowing the conceptual distinctions within a subject domain that are made by subject experts to dominate psychological investigation of that domain" (p. 516). To one familiar with the work and theories of Gelman and Gallisted, Brainerd, Bryant, and others, the research will have even more significance and should be studied in detail. It is a carefully done and reported study.

Since the format was a two-choice situation, the element of guessing could have come into play. Results on 6 of the 7 display types argue against subjects; use of this strategy. It appears that the children had reasons for their choices which could be inferred for 6 of the displays. For the remaining display (C), we can only guess what the reasoning was. Of course it would be nice to know what the children would have given as the reason for their choice, but this is in no way a criticism of the design, conduct, and reporting of the work.

From a mathematics teacher's standpoint the results challenge in some areas such as one-to-one correspondence and comparison. There is also a renewed awareness to the presence and importance of perceptual cues to which children respond.

A question for researchers in mathematics teaching is: What does this psychological research in the area of mathematics imply about how we teach children?



Dreyfus, Tommy and Eisenberg, Theodore. INTUITIONS ON FUNCTIONS. Journal of Experimental Education 52: ?7-85; Winter 1984.

Abstract and comments prepared for I.M.E. by JOHN KOLB, North Carolina State University.

1. Purpose

This study is the first in a series that intends to assess the pre-instructional intuitions that students possess about the mathematical topic of functions.

2. Rationale

Functions are a central topic in mathematics and their complex nature makes it a difficult area for students to learn and teachers to teach. Teaching and learning of functions would be facilitated if the process begins from the students's intuitions. "A concept learned with an intuitive base is likely to be understood by the students and can be built upon in a meaningful way" (p. 78). An important first step is to acquire baseline data on the nature of intuitions that various student populations have about the notion of function prior to formal instruction. Instruction and curriculum materials will be more effective if they can begin from the notions students already hold.

The authors propose a three-dimensional model as a framework for devising and classifying the questions in the assessment. The x-axis of the model, called <u>settings</u>, depicts the various ways that functions can be represented: ordered pairs, arrow diagrams, graphs, tables. The y-axis consists of <u>concepts</u> associated with functions such as zeros, onto, one-to-one, image, preimage, etc. The third dimension corresponds to <u>levels</u> of complexity beginning with finite domain and range with integers to infinite sets, functions of several variables, and implicit and recursive functions. The three-dimensional model contains cells which define the representation of the function, the specific concept,



and the level of complexity. The model provides a basis for formulating questions and guiding the investigation theoretically.

3. Research Design and Procedures

The test consisted of 48 items over the concepts of image, growth, extrema, preimage, one-to-one, domain, and range. Twenty-seven of the items were set in a concrete situation (temperatures at various hours of a given day); the other 21 questions were similar but in an abstract mode (no reference to applications or any units). The test items were incorporated into a booklet in one of three versions corresponding to the way the function was represented: a table, an arrow diagram, or plotted points on a coordinate graph. In every case the functions had finite domain and range. The wording of the questions was carefully chosen to be intuitive and avoid use of terminology associated with functions. In particular, the terms domain, range, function, image, etc. were not used.

The test was administered to 60 males and 67 females in six classes of seventh- and eighth-grade students who had not been exposed to instruction on functions or the concepts in the test. The classes were chosen, two eac., from high, medium, and low mathematical ability levels in Israeli junior high schools. Within each class, one-third of the students received the arrow diagram booklet, one-third received the table booklet, and the remainder got the graph booklets. The test was completed by the students in about 30 minutes on the average.

4. Findings

- a) Righ-ability students performed significantly better (<.05) than the lower-ability students on the overall test.
- b) High-ability students performed significantly better (<.05) than the lower-ability students on the concrete and abstract subtests.



- c) No differences were found between seventh- and eighth-grade students on the test.
- d) High-ability students having the graphing setting scored significantly better than the high-ability students having the table treatment. While no other differences were significant, lower-ability students tended to perform poorest with graphs.
- e) Generally, males scored higher than females on the overall test and the subtests, but the differences were not significant.

5. Interpretations

Since no differences were found between grade levels, it was taken as support for the assertation that the test does not depend upon previous knowledge.

A main conclusion was that high-ability students preferred a graphical approach to the concepts while low-ability students did less well with the graphical representation as opposed to their performance with the tabular and arrow diagram modes.

The theoretical model for aspects of function is to be used as a tool for further studies investigating students' intuitions about functions.

Abstractor's Comments

The investigators have devised an interesting model for analyzing the various aspects of the notion of function. The model helps to enumerate and classify the many facets of the topic that must eventually be mastered by the competent mathematics student. I find it remarkable, however, that of all the concepts that the researchers enumerate for the y-axis for their model, they omit the obvious one of the definition



of a function.

While their model corgists of three dimensions, the factor of concrete vs. abstract setting introduced in the experiment is not included as aspect of the model. All subjects took both parts of the test, abstract and concrete setting, and it appears that this was intended as an important factor in the study. However, the experimenters did not test the significance of the difference of the means on the two subtests for either the whole group or any of the subpopulations. Surely it is an important practical finding to determine if a concrete setting is preferred to an abstract one.

The researchers are interested in the students' intuitions about functions since they believe that they provide a foundation for instruction. They assert that the questions were independent of previous knowledge and cite the 'no difference' between seventh- and eighth-grade students as evidence. Yet the questions they ask concern tables and plotted points on a coordinate graph, clearly knowledge based in elementary school mathematics rather than untutored insight indicative of intuition. If there is an intuition that children bring to functions, it probably resides in the commonplace use of the word function to mean dependence or causality. Children would know from everyday settings that the more gallons of gasoline the pump shows, the more the cost of the fillup. The farther one is from a light source, the less the amount of illumination that reaches you. The influence of one entity on another is a most basic intuition (untutored notion) that a pupil has. Only later, through extensive teaching, is the concept of function formally extended to correspondences between domain and range, where no influence is implied or intended.

As a baseline study, it might be improved by questions that a) assess the students' notions about function as it is commonly used in the language, and b) include as a central baseline concept the definition of function.



Dudley, Alan. THE EFFECTS OF TIME AND DATA PERMUTATION ON THE SOLUTION STRATEGIES OF FIRST GRADE CHILDREN. Focus on Learning Problems in Mathematics 6: 51-78; Winter/Spring 1984.

Abstract and comments prepared for I.M.E. by JUDITH THREADGILL-SOWDER, Northern Illionois University, DeKalb.

1. Purpose

This is a clinical interaction study of problem type, order of data presentation, level of intellectual functioning, and time.

2. Rationale

Other researchers have identified, through clinical inquiry, strategies children use in solving ten distinct addition and subtraction verbal problem types. In this extension of that research, the author examines strategies used on permutations of the original ten problem types, and the effect of time on children at three different ability levels.

3. Research Design and Procedures

Eighteen first-grade children were selected for the study in a manner which allowed three groups of different ability levels (above average, average, and below average) to be formed with three boys and three girls in each group. Five addition strategies (counting all, counting from the smaller number, counting from the larger number, number fact, and heuristic) and six subtraction strategies (separating, separating to, adding on, matching, number fact, and heuristic) were identified, along with uncodeable and inappropriate response categories for each operation. Finally, ten types of verbal problems, three for addition and seven for subtraction, were identified, and six possible permutations of each problem were written. For example, two permutations of a joining addition problem were: "Ralph has 'a' pennies. His father



gave him 'b' more pennies. How many pennies did Ralph have altogether?" and "Ralph's father gave him 'b' pennies. How many pennies would Ralph have if he had 'a' pennies before he got some from his father?" An example of a part-part-whole subtraction problem was: "There are 'c' children in a class. 'a' are boys and the rest are girls. How many girls are in the class?" Addends with sums from 11 to 16 and with an absolute difference of at least 2 were used, excluding doubles. The sixty problems were randomly assigned to six problem sets where each set contained one problem of each type. Each child was asked to solve the problems in each of the six problem sets on six consecutive days at the beginning of the school year, and again six months later. The problems were read to the children and counters were available.

4. Findings

In the initial testing, below-average children had difficulty with almost all the addition problems, but only the addition/comparison problems were difficult for all children. Different ability groups found different permutations to be difficult. On the subtraction problems below-average children again experienced difficulty with all problem types, while the above-average children did well on all problem types and most permutations. The average ability group performance varied according to problem type and permutation. The above-average group found subtraction problems to be easier than addition problems, while addition problems were easier for the other two groups.

On the second testing, addition was easier for the above- and below-average groups, while subtraction was easier for the average group. No permutations appeared easier for the top two groups, while differences were found for the lowest group. The below-average group had only 48% of the problems correct, the average group 80% correct, and the top group 90% correct. The most difficult problem types for both testings were addition/companison (e.g., "Tony has 'a' pennies of candy. Ed has 'b' more pieces than Tony. How many pieces of candy does Ed have?") and



subtraction/joining (e.g., "Anne had 'a' books. At her party she received some more books. Now she has 'b' books altogether. How many books did she receive at the party?"). Children were found to use more sophisticated strategies over time.

5. <u>Interpretations</u>

For addition problem types, children appear to go through several stages: being able to understand the problem, counting all, random counting on, counting on from the larger number, automatic response. Subtraction strategies are not so easily classified as addition strategies, and children appear to use them interchangeably. Some children attempt to directly model the problem, which works for addition but not for subtraction, since addition is commutative and subtraction is not. This has implications for teaching. Children need prerequisite skills before they can successfully solve verbal addition and subtraction problems. This also has implications for teaching.

Abstractor's Comments

This is an example of a very "clean" study. Different ability groups were represented with equal numbers of males and females, the problems were carefully chosen and represented the complete range of possible verbal addition and subtraction problem types with all possible permutations of data. The only thing to go "wrong" was the loss of two students between testings. The author is to be commended for such a well-designed study.

Most of the data were presented in fifteen pages of stem and leaf diagrams, which were difficult to digest, and even these were only "selected" comparisons. I would have preferred to see the data presented in a more palatable manner, or at least a better summary given of the information from the 20 tables.



All in all, I found the article well-written and interesting to read, apart from the fifteen pages mentioned above, and the implications for teaching section, for which little rationale was offered by this study. However, most of the information here will not be new to those who are familiar with other early number research.



Edge, Orlyn P. and Friedberg, Stephen H. FACTORS AFFECTING ACHIEVEMENT IN THE FIRST COURSE IN CALCULUS. <u>Journal of Experimental Education</u> 52: 136-140; Spring 1984.

Abstract and comments prepared for I.M.E. by J. PHILIP SMITH, Southern Connecticut State University.

1. Purpose

The study was designed to identify factors which are significant predictors of success in a first post-secondary calculus course.

2. Rationale

The investigators hypothesize that "an unacceptably high failure and drop rate" in college calculus courses may be significantly improved by developing a reliable procedure to distinguish between calculus-ready and pre-calculus students. The diversity of students' backgrounds, uniformly high secondary school mathematics grades, and a lack of research results concerning the prediction of success in calculus make the matter of good advisement far from a routine task.

3. Research Design and Procedures

The study was conducted in three phases from 1976 to 1980 at Illinois State University. The subjects were students enrolled in Calculus I during fall 1976 (n=235), spring 1978 (n=157), and fall 1980 (n=397). The latter two groups were involved primarily in replicating the investigation involving the first group. The three groups were felt to be comparable along relevant dimensions.

For use with multiple regression techniques, several independent and one dependent variable were identified. Independent variables were five ACT scores (the composite score and scores in mathematics, English, natural science, and social science), high school percentile rank, high



school Algebra II grade, high school grade point average, and the grade on an algebra placement examination. For the 1976 and 1978 groups, sex and major were additional independent variables. For the 1980 group additional independent variables were the score on Rotter's Internal-External Locus of Control Scale, the score on the Group Embedded Figures Test, family size, birth order, high school size, the number of semesters since last taking a mathematics course, and "study time," a self-report on the number of study hours spent per class lecture.

The dependent variable for the 1976 and 1978 studies was a score from 40 to 99 assigned as a grade by the Calculus I instructor. For the 1980 study, only letter grades (including +'s and ~'s) were available. These were converted to a 0 to 4.2 scale and used as values of the dependent variable. A highly standardized syllabus and a common final examination were felt to provide reasonable consistency in grading.

Simple correlations among the variables were examined and variables not correlating significantly with the independent variable were eliminated from further consideration.

A forward stepwise multiple regression procedure was used on the 1976 data to identify the best subset of independent variables for predicting the calculus grade. Several regression models developed on the basis of that data were used to predict calculus grades for the 1978 group. Similar processes were carried out on a random sample consisting of approximately 60% of the 1980 group, and the models so developed were tested on the other 40%.

Findings

Significant correlations (p < .05) were reported between all variables, dependent and independent, except for those noted below. Correlations between Algebra II grades and ACT composite scores, ACT Mathematics scores, and ACT English scores were not significant. No findings of any



sort were reported for ACT scores in natural science and social science. Correlations between sex and major and other variables were not reported, but both of the former were dropped after the 1978 phase as "not significant."

Of correlations involving the dependent variable, the highest (.55 for 1976 data, .61 for 1980 data) involved the algebra placement test score. The various 1976 correlations and those of 1980 are of roughly the same magnitude. Of the additional independent variables considered in 1980, only the number of semesters since last studying mathematics was significantly (p < .05) correlated with the calculus grade. (The coefficient was -.24.)

The multiple regression analysis indicated that the best predictors from the 1976 variables were scores on the algebra placement test and high school rank (R = .524). The addition of a third variable increased R only to .538. Roughly comparable results held for the 1980 data. Several regression models were produced from the 1976 and from the 1980 data. All the best models involved the algebra placement test score and high school rank. Use of only those two variables produced the best 1976 model (R = .60) and, arguably, the best 1980 model (R = .65). Models that included sex and the number of semesters since last studying mathematics as additional independent variables yielded no improvement.

The best 1976 regression model, when applied to the 1978 data, predicted grades whose correlation with actual grades was .71. Further, 84% of the students received a grade at least as high as that predicted by the model. The corresponding statistics for two of the best 1980 models were 74% and 68%, respectively, when applied to the remaining 40% of the 1980 students.

5. Interpretations

The study's main conclusion is that algebraic skills play a



significant role in predicting calculus achievement. An unexpected finding, according to the authors, was the consistent improvement resulting from the addition of high school rank as a predictor. They hypothesize that such a statistic may have a psychological as well as an academic interpretation: perhaps it measures competitiveness as well as some aspect of emotional adjustment in an academic setting. The authors suggest that the increase in the discrete nature of the dependent variable from 1976 to 1980 may account for the less effective predictions of the 1980 models. They felt that a good, conservative placement procedure could be devised on the basis of the 1976 model. Students with predicted grades of C or better are advised to enter calculus; others are urged to begin with a lower-level course.

Abstractor's Comments

In many respects the study is carefully done and its replicatory aspects are commendable. This reader was disappointed, however, by the lack of insights or ponderable issues raised by the research. Most calculus instructors do seem to feel that algebraic skills underlie calculus success, at least initially, and a considerable body of research attests to the importance of high school grades and rank as predictors of college academic achievement. The present study, then, simply seems to nail down our expectations without adding greatly to our understanding of what leads to success in calculus.

In view of the relatively few studies examining the prediction of calculus achievement, one hopes for more than the present study offers. The disappointment, though, is not necessarily the authors' fault: perhaps, in fact, nothing earthshaking is "out there" waiting to be discovered. The additional variables used in the 1980 phase are certainly excellent ones; they preclude the study from being criticized as unimaginative in choosing potential correlates.

The use of alternative analyses of the same data, a search for



quadratic terms in the regression models, and the examination of stability over time are representative of a number of good aspects of the study. The researchers have made fine use of their three groups of subjects, as well, and the replication results give added strength to our confidence in their results.

Some facets of the report, as presented, are puzzling. Three independent variables were identified as such, but never mentioned again. What happened to them? We are given little information about how calculus is taught at Illinois State University. Did the classes involve large lecture sections? How available was help if needed by a student? Potentially, matters such as these affect achievement and, therefore, the generalizability of results. Certain statistics are reported for some phases of the study but not for other apparently comparable phases. Why, for instance, are we given the correlation between predicted and actual calculus grades for the 1976-78 data but not for the 1980 data? Why is there a step-by-step table for the 1980 forward stepwise regression procedure and none for the 1976 analysis? On what basis were choices concerning such inclusions and exclusions made? To be faced with a lack of parallel structure hinders the reader from drawing comparisons.



Fischbein, E. and Gazit, A. DOES THE TEACHING OF PROBABILITY IMPROVE PROBABILISTIC INTUITIONS? Educational Studies in Mathematics 15: 1-24; February 1984.

Abstract and comments prepared for I.M.E. by DOUGLAS E. SCOTT, Amphitheater High School, Tucson, Arizona.

1. Purpose

The primary purpose of the study was to see if a short (12-lesson) course on probability would affect the typical intuitive probabilistic misconceptions of a group of middle-school pupils.

A secondary purpose was to obtain information about the direct instructional effect of the lessons and their suitability for the age level involved.

2. Rationale

The spontaneous probabilistic judgments of people are often biased and incorrect, frequently surfacing as superstition or unsupported opinion. An earlier study had found that instruction in probability actually worsened the performance of some subjects in estimating the chances of obtaining various results. In that study, the best performance was turned in by students with "a high practice in chance games."

The authors' point of view, therefore, is that new intuitive attitudes can be developed only by involving the learner in a practical activity, not by verbal explanations alone.

3. Research Design and Procedures

The subjects were 285 students in grades 5 through 7, with a 305-student control group of like ages. The experimental group was given a 12-lesson course in probability covering such topics as certainty,



impossibility, and probability; counting outcomes; relative frequency; and simple and compound events. The program offered the children the opportunity to be active in calculating probabilities and predicting outcomes using dice, coins, and marbles.

The assessment of the effects of the teaching program was by means of two questionnaires: Questionnaire A was concerned with finding to what extent the notions which were taught had been assimilated correctly; Questionnaire B assessed the indirect effect of the instruction on the pupils' intuitively-based misconceptions.

Questionnaire A included routine items such as "If you coll two dice what is the probability that the sum will be 6? will be 13? will be even?" and some less-common questions like "Referring to the possible outcomes of rolling 2 dice, give two examples of chance events/certain events/compound events."

Examples of items from Questionnaire B are: "Are you more or less likely to win in a lottery if you pick the same numbers each time? or if you vary your picks? or if you repeat a set of numbers that won for you once before? or if you pick consecutive numbers?" and "Which has a better chance of drawing a white marble: picking from a sack with 100 white and 50 black, or 200 white and 100 black?"

4. Findings

From Questionnaire A:

The data show an evident progress with age on all items. About 60-70% of the sixth graders and 80-90% of the seventh graders were able to understand and use correctly most of the concepts which were taught. Most of the notions were, however, too difficult for the fifth graders. They were able to understand the concepts of certain, possible, and impossible events, but not the other concepts -- particularly the ideas of simple and compound events.



Two items were especially difficult for even the seventh graders: the calculation of probabilities of compound events using two dice, and the giving of examples of simple and compound events using one or two dice.

From Questionnaire B:

The results for the fifth graders were unreliable because (as noted above) the direct teaching effects were minimal and thus could hardly have had much effect on intuitive attitudes. Further, many of the younger pupils evidently did not understand the questions, because they either gave no answer or answered at random. The following discussion, therefore, refers only to the results from the older students.

Of the eight groups of questions on Questionnaire B, four referred specifically to common misconceptions: "lucky" numbers, the "recency" effect, and so on. In all these cases, the lessons had a positive effect. Age also had a similar effect (older children performed better in both experimental and control classes), but the experimental group did better than the controls at each level.

The only adverse results were on those items based on proportional reasoning (such as comparing the chance of picking a white marble from a sack with 100 white and 50 black or a sack with 200 white and 100 black), with control groups showing a systematic, though not statistically significant, edge over the experimental groups.

5. Interpretations

The authors conclude that a systematic program on probability can be carried out, possibly starting from grade 6 and certainly from grade 7, and that such a program should have a beneficial effect on the pupils' prejudices and misconceptions with regard to sequences of events in uncertain situations.



A caution is noted with respect to lessons concerning proportional reasoning: the authors suggest that special exercises would need to be devised in which pupils would be confronted with proportional computations and the respective probabilities.

Abstractor's Comments

The lack of success with the fifth graders is important in view of the contin ing interest in the teaching of probability in the schools; certainly this study suggests that there is a realistic lower bound to the age at which probabilistic concepts can be effectively taught. It should be noted, however, that the primary purpose of the study was not to determine what concepts could be taught at what grade level, but rather to determine whether attitudes could be changed as an indirect result of the teaching of probability.

The authors reside in Israel; the study was presumably done there, although it is not explicitly stated; thus, any results should not be generalized to the U.S. without a successful replication. In attempting either a replication or an extension of this investigation, one might consider such questions as: will alteration of intuitive concepts always follow the acquisition of new cognitive learning? would practice with games of chance produce the same results more efficiently? or, if the goal is alteration of intuitive concepts, would it be more effective to use lessons that address those concepts directly, rather than using the indirect method of this study?

Clearly this is a topic that has excellent prospects for further investigation.



Jones, Lyle V.; Burton, Nancy W.; and Davenport, Ernest C., Jr. MONITORING THE MATHEMATICS ACHIEVEMENT OF BLACK STUDENTS. Journal for Research in Mathematics Education 15: 154-164; March 1984.

Abstract and comments prepared by MARTIN L. JOHNSON, University of Maryland.

1. Purpose

To analyze in detail the average white-black difference in mathematics achievement on the 1975-76 "selected mathematics study", a part of the National Assessment of Educational Progress (NAEP), and to explore the relation of mathematics performance to characteristics of schools and of individual students.

2. Rationale

Interpretation of achievement differences between white and black students over the two assessment years is complicated due to testing procedures, changes in student population, and reporting procedures. More understanding will result from a detailed analysis of the data from a single assessment.

3. Research Design and Procedures

Data were collected on mathematics performance and other selected variables from the 1975-76 selected mathematics study. From these data, means and standard deviations of mathematics scores for all black females, black males, white females and white males were determined. Also, data on background and school variables were collected.

4. Findings of the Background Variables

Regression analysis suggests that at age 17 the number of high school courses taken in algebra and geometry, high school grade point



average, and grade in school are the best predictors. Among school variables, important predictors were the mean number of reading materials in the home, mean number of mathematics courses taken, parental occupation level of the school, mean grade point average, mean parental education, and mean grade. Black-white differences in background variables account for about half of the white-black mean differences in mathematics achievement scores.

5. <u>Interpretations</u>

Since taking first-year algebra, second-year algebra, and geometry appear to be the best predictor of achievement at age 17, it becomes imperative that enrollment in mathematics courses be encouraged. School-to-school differences as well as white-black differences would be reduced if mathematics enrollments in predominately black schools could be increased.

Abstractor's Comments

This study provides an interesting analysis of mathematics achievement data in an attempt to explain racial differences in performance. The findings reinforce what other researchers have found: a direct relationship between courses taken and mathematics achievement. White students enroll in advanced classes more frequently than black students and hence their performance is higher. While interesting, this is hardly new information. Perhaps we can now begin to find ways to address the main issue: encouraging students of all races to take more mathematics!!!



Kail, Robert; Stevenson, Michael R.; and Black, Katherine N. ABSENCE OF A SEX DIFFERENCE IN ALGORITHMS FOR SPATIAL PROBLEM SOLVING. Intelligence 8: 37-46; January-March 1984.

Abstract and comments prepared for I.M.E. by JOE GAROFALO, State University of New York at Buffalo.

1. Purpose

The purpose of this study "was to determine if there is a sex difference in use of algorithms for solving one common type of spatial problem" (p. 37). The specific problem in question is one where the subject is first presented with "two versions of a letterlike stimulus," one upright and one rotated 0-150 degrees from the vertical, and then is asked to "determine if the two stimuli would be identical or mirror-images when they are rotated to the same orientation" (p. 37).

2. Rationale

Cooper and Shepard (1973) proposed that four processes are involved in solving such spatial problems: encoding the identity and orientation of the standard (upright) stimulus, rotating the representation of the comparison (rotated) stimulus to the orientation of the standard stimulus, comparing the rotated representation of the comparison stimulus with the standard stimulus, and responding "same" if they are identical or "different" if they are mirror-images. Two variations on this algorithm are: (a) only rotating the representation of the comparison stimulus when irs angle of rotation is greater than a critical angle from the vertical; and (b) on mirror-image pairs, waiting until a time deadline is reached before responding "different." Thus, there are four possible algorithms for solving such problems since there are two alternatives regarding the processing of mirror-image pairs (responding immediately or after a deadline) and two alternatives regarding the rotation of the comparison stimuli (obligatory or contingent upon the angle of orientation). The objective of the study was to determine the extent to



which males and females differ in the use of these four algorithms.

3. Research Design and Procedures

For each of five letterlike stimuli, 20 slides were prepared depicting both a standard (upright) and a comparison (rotated) stimulus. The standard and comparison stimuli were identical on ten of the slides and were mirror-images on ten. Under each condition, the comparison stimulus was rotated clockwise 0, 15, 30, 45, 60, 75, 90, 105, 120, and 135 degrees from the standard. The 100 slides were presented twice, in randomly ordered blocks of 20, subject to the constraint that each combination of type and orientation appear once per block. The slides were projected onto a screen and were viewed by 36 male and 32 female undergraduates, who responded "same" or "different" by pressing one of two telegraph keys.

4. Findings

- A. Errors. Errors increased as a function of the orientation of the comparison stimulus (p < .01), and were more likely on identical pairs than on mirror-image pairs (p < .05). The increase in errors as a function of orientation was more pronounced for identical pairs, resulting in a significant interaction (p < .01). There was no significant difference in overall error rate for males (6.8%) and females (6.6%).
- B. Response Latencies. For each combination of orientation and type, a median response time was calculated based on correct responses. Main effects included: (1) males responded more rapidly than females (p < .01); (2) identical pairs were judged more rapidly than mirror-image pairs (p < .01); and (3) latencies increased as a function of the orientation of the comparison stimuli (p < .01). Interactions included: (1) latencies increased more rapidly as a function of orientation on identical pairs than on mirror-image pairs; and (2) the increase in latencies as a function of orientation was similar for males and females



on mirror-image pairs, but was more rapid for females on identical pairs.

C. Evaluation of Models. The pattern of latency data across subjects did not correspond to any of the patterns predicted by the four models, so a best-fitting model was identified for each subject. STEPIT (Chandler, 1965) was used to find parameter values that minimized the squared differences between observed latencies and those predicted by the models. Sixty-four subjects (94%) were successfully fit to at least one model, but typically more than one model accounted for a significant portion of variance. A model was considered best if it accounted for at least 5% more variance than the next best. (Actually, the best ones accounted for an average of 20% additional variance.)

Using this criterion, the best-fitting models accounted for an average of 72% of the variance in latency data, with a range of 47% - 90%.

Thirty-four (50%) subjects were identified with the model involving obligatory rotation and an undelayed response of "different;" ten subjects were identified with obligatory rotation and the processing of mirror-image pairs until a deadline; nine subjects were fitted to the model that involved an immediate response on mirror-image pairs and rotation of the comparison stimuli only when its orientation exceeded a critical value ($\mathfrak{F} = 74$); eight of the nine subjects whose data fit two models were identified with using obligatory rotation; four subjects were not identified with any model. If the criteria for the best-fitting model is raised to 10% additional variance, ten more subjects would have been identified with two models. There were no sex differences in any of these results.

5. Interpretations

The authors concluded that males and females "did not differ in the frequency with which they used various algorithms to solve the type of spatial problem presented in this study" (p. 44). They did acknowledge that this finding applies only to spatial tasks involving rotation in



the plane. They offered a potential objection to their conclusions; namely, that sex differences could actually occur in algorithms other than those tested in the study. They discounted this objection on the grounds that other alternative algorithms are not specific enough to derive response time predictions to be tested, and that since the best-fitting models accounted for an average of 72% of the variance, they muse not be radically different from the "true" models.

The authors offered a possible explanation for the sex differences in the rate of mental rotation of identical pairs. Referring to data that suggest that mental rotation is actually accomplished in "steps," they hypothesized that perhaps the step size is larger for males; this would allow them to solve problems faster.

The tendency to process to a deadline was linked to poor encoding of stimuli. Imprecise or degraded codes are more likely to make identical pairs appear different than to make mirror-image pairs appear identical. This asymmetry "may bias subjects to continue processing mirror-image pairs to the deadline" (p. 45).

Finally, a possible explanation of contingent rotation was offered. During the course of the trials, some subjects may have stored representations of the stimuli, at a variety of orientations, in long-term memory. This would "obviate the need for mental rotation in some instances" (p. 45).

Abstractor's Comments

Working under the premise that sex differences in performance on some spatial tasks may be due to differences in algorithm usage, the authors of this straightforward study set out to determine the degree to which males and females differed in the use of four proposed algorithms to solve a specific spatial comparison problem. They have demonstrated, to their satisfaction, that the pattern of usage of the



four algorithms was similar for males and females. The only sex differences found were in response latencies. The authors also offered explanations for the distinguishing features of the algorithms. I will now state some of my concerns about the study and the authors' interpretations of findings.

As the results clearly indicate, the fitting of algorithms to subjects was far from perfect. The authors concluded that since the best-fitting models accounted for an average of 72% of the variance, they were probably close to the "true models." But since the range of variance accounted for was 47% - 90%, it is obvious that some of the best-fitting models fit poorly and, therefore, may not be "true." No information was given pertaining to how many of the fits were poor and how the poor fits were distributed across the four models and across the sexes. Was it the case that the contingent rotation models had poorer fits than the others? Was it the case that males and females had an equal percentage of poor fits? I would want to have such information. One way of explaining the poor fits is that maybe some subjects were using an algorithm different from those proposed, or some other variant of the Cooper-Shepard model. The authors discounted this possibility (see Interpretations). I find the authors' reasons for this unconvincing.

The authors have implicitly assumed that the use of a specific algorithm was consistent over trials. They have not allowed for the possibility that a subject might use more than one algorithm even for the same or similar comparisons. Given that some subjects were poorly fit to a model, that others were fit to more than one model, and that still others were not fit to any model, I think this assumption is questionable.

Finally, I would like to have seen some more data and discussion concerning error rates. I would be interested to see if there was any relationship between choice of algorithm and rate of errors. Did obligatory lotation yield better results than contingent rotation? Did



waiting to a deadline on mirror-image pairs produce better results than responding immediately? I would also be interested in the authors' views on the relationship between error rates and response latencies. Both increased as a function of the orientation of the comparison stimulus, expecially for identical pairs, but the higher error rate for identical pairs is coupled with lower response latencies.

In general I found the study interesting, but would have preferred to see more data broken down by choice of algorithm.

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Marshall, Sandra P. SEX DIFFERENCES IN CHILDREN'S MATHEMATICS ACHIEVEMENT: SOLVING COMPUTATIONS AND STORY PROBLEMS. <u>Journal of Educational Psychology</u> 76: 194-204; April 1984.

Abstract and comments prepared for I.M.E. by CAROL NOVILLIS LARSON, University of Arizona, Tucson, Arizona.

1. Purpose

This study examined sixth-grade girls' and boys' performance on computation and story problem standardized test items to determine if there were sex differences. It also investigated whether the following four factors interact with sex to influence mathematics performance: reading, socioeconomic status (SES), primary language spoken at home, and chronological age.

2. Rationale

The results of several previous studies comparing males' and females' mathematics achievement have not been consistently in favor of one sex or the other. In order to make progress in understanding these mixed results, the author focused on comparing boys' and girls' performance on two well-defined types of items: computations and story problems.

A relationship between reading and successful solving of story problems has been previously reported. Since girls generally score higher than boys on reading tests the following question arises: does there exist a Sex by Reading Score interaction that predicts successful solving of computations or story problems? Other factors—SES, language, and age—have also demonstrated test score differences. These were also investigated for interactions between each factor and sex on the two types of items.



3. Research Design and Procedures

The subjects were 286,767 sixth graders who completed, in 1979, the Survey of Basic Skills, a standardized test developed by the California Assessment Program. The test has 16 forms, each one containing 30 distinct items. In this study only the eight reading items and some of the 10 mathematics items on each form were considered. The mathematics items analyzed were the computation items and the word problem items involving fraction, whole number, and decimal arithmetic. Each form of the test contained four or six relevant mathematics test items. The total set of mathematics items analyzed was 41 computation items and 27 story problem items.

Data analysis consisted of two parts: 1) a log-linear analysis of the data from 15 test forms to determine whether there were sex differences on the two types of items; and 2) the analysis of the data from eight test forms to investigate the interaction of sex with the other four factors. For the log-linear analysis, each student's responses to both types of items were converted to two n-tuples of 0's and 1's called response patterns. An incorrect score was coded as 0 and a correct score as 1. This resulted in "a set of cross-classified categorical data over three factors: response patterns for computations, response patterns for story problems, and sex" (p. 195). The log-linear analysis was performed on the response patterns rather than on the number of correct responses to each item type. This analysis consisted of comparing the data from each of the 15 test forms with the expected frequencies that occur under all possible hierarchical log-linear models.

To analyze the influence of reading, SES, language, and agr, only the eight test forms containing two computation items and two word problem items were used. The sample was then further restricted to those students successful on at least one item type—this was defined as responding correctly to the two items of the same type. For each of the four factors, the probability of answering correctly both computations and/or story



problems was reported for each level of the factor. A discrepancy score was then computed for each level of a factor by subtracting the probability of success on story problems from the probability of success on computations. For each factor, the regression lines showing the relationship of the factor and the discrepancy between solving computations and story problems were plotted and the difference between the two regression coefficients tested.

4. Findings

Log-linear analysis. For all but one test form, the only satisfactory model was the model of main effects and all two-factor interactions. This model of best fit had probability levels for the different test forms ranging from .074 to .846, with a mean of .332. All other models were rejected with p < .001. The direction of the interactions in the model of test fit for all 15 test forms were: 1) "successful solving of computations was positively associated with successful solving of story problems" (p. 197); 2) girls were more successful than boys in solving computations; and 3) boys were more successful than girls in solving word problems.

Additional factors. For reading, SES, language, and age, girls were more successful on computation items and boys on story problem items for most levels of these classifications. For each of these four factors, sixth-grade students were more successful in solving computations than in solving word problems.

For both sexes the discrepancy between computation ability and story problem ability increased as reading scores increased. The discrepancy for girls increased at a significantly greater rate than did that for boys. The regression coefficients were .009 and .021 for girls with t(14) = 3.67, p < .01.

There was little or no interaction between SES and sex, or between



language and sex. On both types of items, performance of older-than-normal sixth-grade boys and girls declined. For students progressing normally through elementary school (138-150 months), age was not a factor that contributed to sex differences in mathematics achievement.

5. <u>Interpretations</u>

The author concludes that this study supports the theory of differential abilities of boys and girls over two specific item types. Similar differences have been reported for high school students. These results indicate that these differences also occur with sixth graders. This finding may also help explain the mixed results of previous studies. The type of items used on the tests in these studies might account for their results. Other item types should also be investigated for sex differences that may exist.

In discussing the surprising reading results, the author suggested that perhaps girls are more rule-governed than boys, who are perhaps more flexible in approaching word problems. This was suggested as an area of further study.

Abstractor's Comments

A great deal of progress has been made in the last decade in identifying why females have traditionally avoided the study of mathematics. This research has motivated various types of programs for bringing about change in this state of affairs. The current study focuses on whether girls and boys differ in their ability to solve two types of mathematics problems: computations and story problems. Even though sophisticated statistical analysis is used and significant results of sex by type of test item is reported, more research in this area with a different testing instrument needs to take place. The nature of the test is very unsatisfying in addressing this question.



The author's main question focuses on type of test item--computation or story problems--with the items extending over whole numbers, fractions, and decimals. Each student responded to a total of four or six items. Thirteen forms of the test contained only four items--eight of those forms had two of each type of item; the other five had at least one of each item. Two of the test forms had six items; the number of each type of item is not included in the article. This does not seem to me to be an adequate sample of test items to use in exploring such an important question. I would like to see this question re-examined with a testing instrument which contained more items and systematically tested computations and story problems with whole numbers, fractions, and decimals.

The report of the log-linear analysis could have contained more information, such as some tables of observed patterns and chi square models with probabilities and degrees of freedom.

The analysis used to answer the second question addressed by this article—"Are there additional factors (e.g., reading achievement or socioeconomic status) that interact with sex to influence mathematics performance?"—is very unusual. The major problem is that the statistical analysis used is so far removed from the students' scores that the results are very difficult to interpret. What is tested is the difference between two regression coefficients where the regression lines predict "discrepancies" between computation and story problem success from the level of the factor (e.g., reading score) for each sex. These discrepancies are based on the percentage of students of each sex who correctly responded to two computation test items and/or two story problem test items. I would like to see the mean number correct for each type of item for each sex at each level of the factor.

The reading test which is used for testing whether there is a sex by reading interaction contained only eight items. A description of the items is not included. Since reading items could test many aspects



of a reading program such as phonics, vocabulary, or reading comprehension, the type of items included might help in understanding the reading result.

This study posed interesting questions that need to be further explored with different instruments and, in the case of the second question, a different type of analysis.



Rubenstein, Rheta N. COMPUTATIONAL ESTIMATION AND RELATED MATHEMATICAL SKILLS. Journal for Research in Mathematics Education 16: 106-119; March 1985.

Abstract and comments prepared for I.M.E. by CAROL A. THORNTON and JUDY A. CARR, Illinois State University.

1. Purpose

The three purposes of this study were (a) to develop instruments to measure four types of computational estimation tasks and several related mathematical skills, (b) to assess computational estimation performance within several dimensions, and (c) to explore the relationships among the computational estimation tasks and the mathematical skills.

2. Rationale

Although mathematics educators generally agree that estimation is an essential skill, research has shown that the instruction of estimation is often neglected and that students of all age groups are weak in estimation skills.

Previous studies have hypothesized various prerequisites for successful computational estimation. Among these prerequisites are the ability to compute single-digit arithmetic, the ability to work with rounded numbers, an understanding of order relations, and an understanding of the nature of the decimal number system and its processes.

Research on estimation and its related mathematical skills is difficult because various meanings are associated with computational estimation and traditional standardized timed tests do not validly measure estimation ability.



3. Research Design and Procedures

Four hundred thirty-five average to high-achieving eighth graders in middle- and upper-class Michigan communities were tested in this study. Of these students, 309 composed the sample. An experienced eighth-grade mathematics teacher administered the tests during three instructional periods.

Four types of computational estimation skills were measured on the 64-item Estimation Test. The test consisted of four 16-item scales, one for each type of task:

- (1) The Open Ended Estimation Scale gave no answer choice; acceptable responses fell in a predetermined interval.
- (2) The Reasonable vs. Unreasonable Estimation Scale required the subject to decide whether a calculator-displayed answer was reasonable or not.
- (3) The Reference Number Estimation Scale required the subject to decide if an exact answer was larger or smaller than a given reference number.
- (4) The Order of Magnitude Estimation Scale presented four choices of different powers of ten; the subject had to choose the one closest to the exact answer.

Each item on the Estimation Test was slown on an overhead projector for 15 seconds, with five seconds between each item for students to record an answer and for the teacher to replace the transparency.

In addition to the Estimation Test, an 80-item Related Factors Test was developed to measure eight mathematical skills believed to be related to computational estimation: (1) selecting operations; (2) making



comparisions; (3) knowing number facts; (4) operating with tens; (5) operating with multiples of ten; (6) knowing place value; (7) rounding; and (8) judging relative size. Subjects were given 18 minutes to complete this test.

4. Findings

On the Estimation Test, the only partition with an unacceptable reliability estimate was the Reasonable vs. Unreasonable. Because of its weaknesses, it was eliminated from further analysis as a separate variable, but kept as part of the total estimation score.

A series of analyses of variance tests (ANOVA) revealed that in the type, form, number, and operation dimensions the overall mean for boys was higher than that for girls. For the three scales taken separately, however, the difference between means was significant only for the Order of Magnitude Scale, with the boys' mean higher.

On the type dimensions, the order of difficulty of the three scales from hard to easy was Open-ended, Reference Number, and Order of Magnitude. On the numbers dimension, decimal numbers were more difficult than addition and subtraction, and division was more difficult than multiplication.

From the Estimation Test results, a multiple regression analysis indicated that 46% of the variance in the test scores was attributed to the Operating with Tens Scale, the Comparisons Scale, and the Getting to Know the Problem Scale. The Operation with Tens Scale, which was the most difficult scale on the Related Factors Test, accounted for most of the variance in predicting performance on the Open Ended and Order of Magnitude Scales.



5. Interpretations

It was concluded that the partitionings within various dimensions reveal differences in computational estimation performance. Order of Magnitude tasks were the easiest for students and Open Ended tasks were the most difficult.

Estimation tasks in numerical form were not more difficult than those in verbal form, and estimation involving decimals was more difficult than estimation with whole numbers. The order of difficulty for estimation of operations reflects the conceptual difficulty these operations present, as reflected in the order they are treated in the curriculum. Division and multiplication also have greater effects on the Order of Magnitude (an important element of a good estimate) than do addition and subtraction.

In terms of sex differences in computational estimation, boys scored higher than girls. However, the differences were most pronounced in the Order of Magnitude estimates, where the difference was only about one item out of 16.

The mathematical skills that contributed most to the prediction of estimation performance were operating with tens, making comparisons, and judging relative size. Operating with tens had an especially strong relationship with estimation performance. Students lacking the skills of multiplying or dividing by powers of ten have a diminished understanding of the size of a number which is considered to be essential for estimation. This skill also involves multiplication and division, as do the more difficult estimation tasks. The fact that the seemingly simple skill "sliding the decimal point" is apparently so significant suggests that educators should be looking for better ways for students to develop and apply it.

Skills currently stressed in textbooks and that one might have



predicted would be important ior estimation (knowing place value, knowing number facts, rounding, and operating with multiples of ten) did not explain more than 1% of the variance. They may be necessary skills, but other skills contribute more.

Abstractor's Comments

This status study was well-designed and carefully carried out, and addresses a matter of critical importance in today's elementary school curriculum. The investigation provides useful information to teachers by extending other recent research and curricular efforts related to computational estimation.

The practical significance of the study lies in its identifying "operating by tens" (i.e., multiplying and dividing by powers of 10) as a high predictor of success in computational estimation. The implication for curriculum development and classroom teaching is clear. The suggestions made by the researcher for further study deserve serious consideration.



Threadgill-Sowder, Judith. COMPUTATIONAL ESTIMATION PROCEDURES OF SCHOOL CHILDREN. <u>Journal of Educational Research</u> 77: 332-336; July/August 1984.

Abstract and comments prepared for I.M.E. by ROBERT D. BECHTEL, Purdue University Calumet.

1. Purpose

The purpose of this study was to obtain information on the cognitive processes used by school-age children when confronted with problems requiring estimation.

2. Rationale

Strong support for developing estimation skills has been given by professional organizations, including the National Council of Supervisors of Mathematics, the National Council of Teachers of Mathematics, and the Conference Board of the Mathematical Sciences. Children are not learning these skills. There has been little research directed towards the development of estimation skills and causes of difficulties with estimation. Information obtained in this study could be of value to researchers interested in studying the development of estimation skills and concepts, and to teachers who desire a better understanding of difficulties students encounter when estimating.

3. Research Design and Procedures

The sample was comprised of 26 students from grades six through nine, ranging in age from 11 to 15. Twelve computational estimation problems, representative of the types of items used on NAEP tests, were selected for the interview list. Half of the problems were presented in "story problem" format. "Students were presented the problems and asked to work them aloud, explaining how they obtained each answer. They were not allowed to use any writing materials. Their responses



were tape recorded and later transcribed."

4. Results

All of the problems given to the students are shown in the article. For each multiple-choice problem the percent of students responding to each choice is given, and a summary is given for each of the other problems. Examples of acceptable and unacceptable explanations are also included. The information given for problem #7 serves to illustrate this section of the article.

Prob. 7. A quart of water weighs about 2.1 lbs. About how much do 13.8 quarts of water weigh? The answer is closest to:

6 1/2 (12%)

7 (4%)

26 (15%)

28 (62%, with 46% acceptable)

I don't know (8%)

5. <u>Interpretations</u>

The problems were NAEP-type items. Some students gave correct answers, but unacceptable explanations. This suggests that the results from the NAEP studies are probably even worse than reported.

Doing problems without paper and pencil was difficult for some students to accept. Many students were more comfortable with direct computation than with estimation. Many of these same students said they "didn't know what to do" with story problems. Several students equated guessing at an answer with estimating. "The lack of 'number sense' exhibited by many of these students, along with their inability to correctly use rounding off procedures and their desire to find 'rules' to guide them, lend support to the recent call by the Conference Board of the Mathematical Sciences for 'heavy and continuing emphasis on



estimation and approximation, not only in formal round-off procedures, but in developing a feel for numbers."

Abstractor's Comments

The researcher used an approach of interviewing a few students about using estimation in problem solving. As noted in the article, students are in need of help in developing estimation skills. Preliminary, informal studies of this type are useful in assessing the problem. Behaviors of students need to be recorded and examined. Questions need to be raised.

The researcher raises some interesting points. She notes that some mathematics educators have argued that a "number sense" or quantitative intuition is required for estimation skill. She states that "perhaps estimation skill and number sense are too intimately entwined to separate them and teach one without also teaching the other."

This abstractor would suggest pursuing the analysis of the problem further. Terms such as "number sense" need to be replaced by specific behaviors. Does a child have a sense of magnitude for numbers (measures) in real-world (i.e., child real-world) settings? How does a child respond to choices that vary by powers of ten? For example, a family of four eats at a fast-food restaurant. The total bill for the family would be about: a) \$10.00 b) \$1.00 c) \$100 d) \$0.10.

Sound estimation procedures for operations on whole numbers and with decimals (these procedures are available) need to be understood by teachers and taught to students. Mental arithmetic is a related topic that has a role. And using the problem-solving technique of proposing a simpler problem (with 'easy' numbers) is another facet. All of these topics need to be developed as individual skills and then as an integrated package.



van den Brink, F. J. NUMBERS IN CONTEXTUAL FRAMEWORKS. Educational Studies in Mathematics 15: 239-257; August 1984.

Abstract and comments prepared for I.M.E. by DOUGLAS EDGE, University of Western Ontario.

1. Purpose

This study is a report on contextual influences when introducing addition and subtraction to children in kindergarten and grade one. Focus is given to a bus context (boarding and leaving).

2. Rationale

The author noted that in traditional mathematics education, contexts are purely illustrative. These contexts, however, have often not been experienced by children. Children require a concrete basis for understanding number. By concrete is meant the "context or situation with which a child is, or shall be, familiar" (p. 241). It is only the accumulation of experiences within various contexts that permits abstraction (i.e., conscious avoidance of contextual frills). Hence the author stated that contexts must be examined to see what limitations they impose upon context-free numbers and operations. What number properties may operate within one context but not another? Are some contexts more realistic than others for certain children? Do teachers make certain assumptions about children's knowledge of contextual situations?

3. Research Design and Procedures

Firstly, although the author mentioned situations involving skittles, magic numbers, and so on, what was meant by the bus-context was described in some detail. Three numbers are important: the starter (the number of passengers on the bus before the bus-stop); the operator (a + or - sign followed by a number to represent passengers boarding or leaving),



and the resultant (the number of passengers on the bus after the bus leaves its stop). An 'arrow-notation' is used to record the action: a starting number and arrow (with operator) pointing to the resultant. The author indicated that these "bus numbers" have specific characteristics such as non-negative, are directional in reading, and allow for simultaneous addition and subtraction. The characteristics impose certain limitations on the numbers involved. "That is the essence of this contribution" (p. 245).

The author then proceeded to discuss the phases of his research. In the first phase "we talked with eight or ten children from the same kindergarten or first grade class" (p. 246) using a method of "mutual observation." In the second phase the author wrote that it was discovered that children had made specific mistakes and had misconceptions about the bus model. Hence suggestions were made to form an experimental kindergarten or grade one class to accentuate "the play element in the context of arithmetic education" (p. 246). The final phase compared the instruction in the experimental and traditional mathematics classes.

Within the article conversations with several students are reported.

4. Findings

From children's responses to questions and reactions to different situations the author made numerous observations: Bus stops appear to be less important than the eventual destination for kindergarten children, but not for children in grade one. Children create appropriate bus structures that differ from those of the adult world. There are contexts (such as toy trains) that have structures that match more closely those of mathematics. The 'direction in which we read' versus 'the direction in which the bus may be travelling' may cause some confusion. And, kindergarten children tend to regard movement per drawing whereas those in grade one more likely see movement per series of drawings.



The author also identified several misconceptions regarding the 'graphic bus'. Examples include the following: Children often believe the bus is either filled or empty when arriving at a bus stop. Children may confuse the resultant with the operator (if three board a bus making a total of six, did three get on or did six get on?). Children must recognize that adding and subtracting may occur simultaneously at any one bus stop and that this may not be allowed in other contexts (for example, in skittles, pins may get knocked down but rarely do others pop back up). And finally, children make errors in doing the sum (if two passengers were on the bus (starter number), and five were on the bus after it left the bus stop (resultant number), many children would write using the arrow-notation that seven passengers got on (operator number)).

In the case of the last mentioned example, the author indicated that children did not make those errors during the bus 'games' but only after the notation was introduced.

5. Interpretations

As contexts influence the use of numbers and operations, contexts should first be presented with situations and then numbers discussed within that contextual framework. Only later should comparisons among contexts occur.

Play acting should be an integral part of teaching arithmetic. Results of this acting should be recorded using arrow-notation.

Abstractor's Comments

It is most distressing to recognize that selected reviewers and an editor could have agreed to publish an article in such a condition.

Admittedly, the concern that mathematics teachers should be aware



that contexts in which examples, illustrations, and problems are presented will likely influence children's learning of mathematics is relevant. Further, the attempt to study this concern using interview or 'mutual observation' strategies is very appropriate. However, the reporting of this research, where one expects to find a clearly stated purpose, and associated rationale, and a well-reasoned, well-described methodology, is totally unacceptable.

Specifically, consider the following:

1. The closest the author comes to a statement of purpose is:

Our present investigation concerns a comparative study of this instruction, as it is given at the experimental school, and traditional mathematics instruction at the control school (p. 246).

Nowhere again throughout the entire article is there any reference to what might have occurred in the traditional program. Further, there is only passing reference to errors that children began making part—way through the year in the experimental program. What were these programs like? Is it appropriate to interview children in the experimental class to address the authors' stated concern with comparative instruction? On what basis were classes and children within classes chosen? Is this study part of a much larger study? Why was the bus context singled out for specific attention?

2. Without a clear understanding of what the author was trying to study, it is somewhat difficult to comment on methodology. However, when the author begins his description with "we interviewed 8 or 10 children ..." (p. 246) one becomes wary. Doesn't he know himself? Reporting throughout the article is often poorly organized, and does contain certain apparent errors. In a section on hypotheses



relating to children in grade one, he refers to two children, Natali and Erik (both five). These were the same two children he referenced when he was discussing children in kindergarten (see p. 246 and p. 252).

- 3. It is difficult to identify statements that represent conclusions based on a rationale and ensuing investigation. The author merely reiterated as a conclusion what must be considered a major point for the rationale for this study: context in which numbers and operations are used matters. Other "conclusions" are presented, yet they don't seem to follow from any investigative aspect of the study. For example, the author appeared to conclude that play-acting must be an integral part of an arithmetic course. That may or may not be true. Did the experimental class do play-acting and the traditional class not?
- 4. The article is also poorly written {and edited}. There is extensive use of "etc.". There is also constant repetition of certain terms such as "skittles". Within the article, aside from comments regarding grade one being written in the kindergarten sections and vice versa, the author is often disorganized. At one point he wrote 'we first wished to find out ..." (p. 246). Nowhere do we find a second or third point!

It is quite possible the author does have some valuable information to share with the mathematics education community. However, one cannot, in good conscience, accept conclusions or recommendations made in this article. The reviewers and editor should have returned the original manuscript to the author with recommendations for extensive revision. At a minimum, what is needed is a statement of purpose, a description of methodology (to include sample, protocols, limitations, and so on), and presentation of findings, with some discussion of the relevance of these findings.



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